DECODING THE SPITFIRE

PART 2

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Introduction

In this second part of the Spitfire study, we try to cover elliptical wings in such a way that non-technical readers can understand the concept. Although the aircraft engineer will consider many of the properties of the elliptical wing, only those necessary for this study will be covered.

The equations used will be easy to follow as only high school math is needed and a simple relation for the wing shape is given. Derivation of the equations is given in the Appendix for those who would like to how this was done, or interested in designing their own shapes.

Plane figures of constant area

A two dimensional shape may be represented by a closed border defining its area. This area may be on a curved surface, but in this study we shall consider only shapes on flat surfaces.

Under certain conditions the area can be kept constant by changing the border shape. This can be achieved by varying the border in a single dimension and in a specific way. A simple example is that the area of a parallelogram is the same as that of a rectangle with the same base and height (Fig. 1).

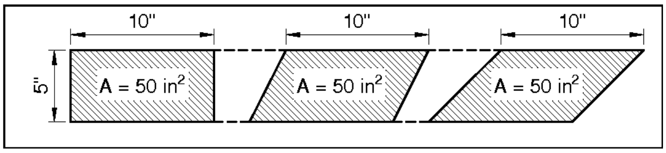

Figure 1

Without attempting a full-proof definition this can easily be demonstrated if an area is divided into strips of smaller areas that are able to slide along one axis. A tool called a contour gauge and used in woodwork (and architecture) can be used to demonstrate this (Fig. 2).

Figure 2

A typical contour gauge consists of a set of pins that are set in a frame which keeps them parallel while allowing them to move independently, perpendicularly to the frame. When pressed against an object, the pins conform to the shape of the object and can then be used to draw the profile or to copy it on to another surface.

Figure 3 is a drawing of a contour gauge.

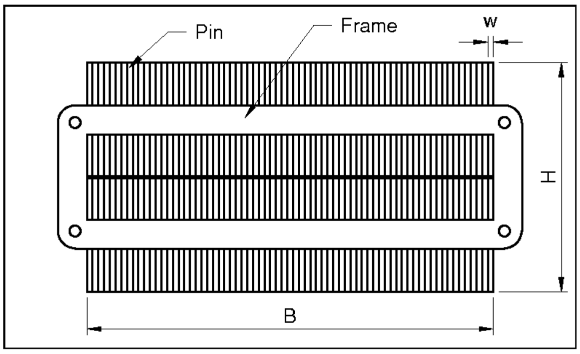

Figure 3

It can easily be appreciated that no matter how the pins are arranged the total area is the sum of each individual pin's area and therefore does not change. This will hold true as long as the pins' sides are in contact with each other (Fig. 4).

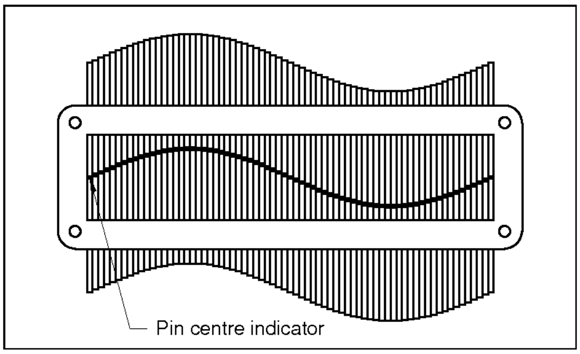

Figure 4

For the sake of simplicity the contour gauge analogy will be used with the frame omitted (Fig. 5).

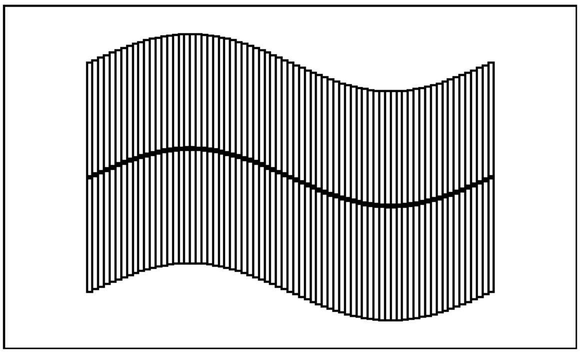

Figure 5

The above also holds true when the pins are not all equal in length but vary along the width of the frame. Such an arrangement is shown in Fig. 6 where the pins represent the shape of a circle.

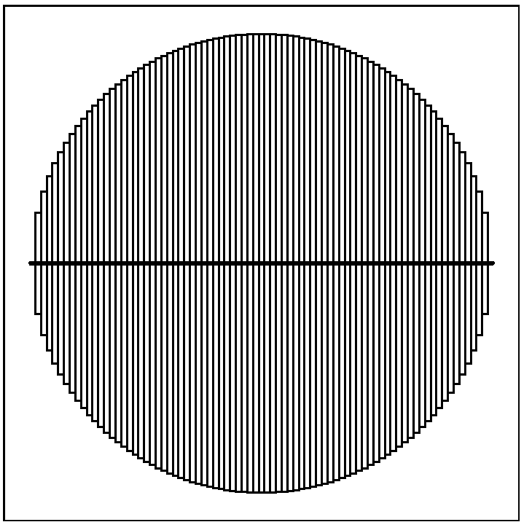

Figure 6

Elliptical Wings

Simply stated, an ellipse is a squashed circle. The degree of compression is called the aspect ratio (AR)[1] of the ellipse and is the ratio of the minor axis to the major axis. Two extreme examples are the circle with an AR of unity and a straight line of length of the major axis with an AR of zero.

I practical everyday life, the ellipse is much more common than a circle as it is rarely looked at squarely, as circular objects are usually seen at an angle and therefore representing an ellipse.

This compression of a circle may be also be described as 'unproportioned scaling', or scaling in one axis. IF the pins in Fig. 6 are split in half and their midpoints arranged in a straight line the resulting outline will be an ellipse (Fig. 7).

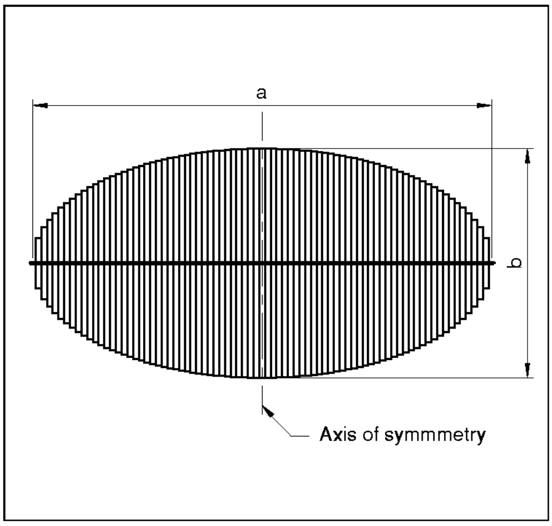

Figure 7

In this case the AR will equal to b/a, or ½. Each pin must be scaled by the same factor, in this case ½.

[1] Not to be confused with Wing Aspect Ratio used in aeronautical engineering and is the wing-span squared divided by wing area ($\lambda = b^2/S$).

Since the ellipse in Fig. 7 is symmetrical about the vertical axis we may therefore consider only one-half. Such semi-ellipse is shown in Fig. 8 and may represent an aircraft wing with a so called 'elliptic chord distribution.

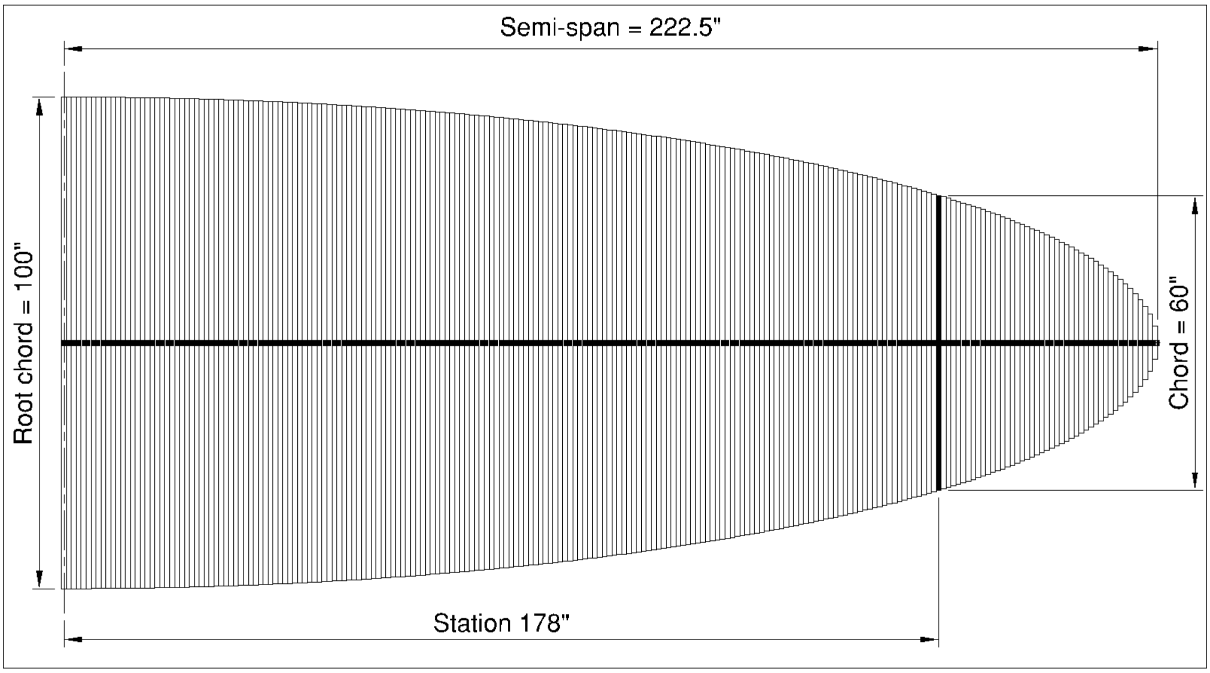

Figure 8

The local wing chord varies elliptically along the span and reduces to zero length at the tip. In Fig. 8 the chord length at Station 178" from the root chord is 60", as an example.

The following equation defines the chord length at any station:

$$C = 100\sqrt{1-4\left(\frac{x}{445}\right)^2}$$

Equation 1

Where x is the distance from root chord.

Wing area is given by

$$S = \pi \cdot \frac{100 \cdot 445}{4 \cdot 12^2}$$

Equation 2

And equals 242.71 ft^2.

Just as done previously, we can now slide the 'pins' (or chords, in this case represented by 1" wide strips) and change the wing plan-from without altering its area.

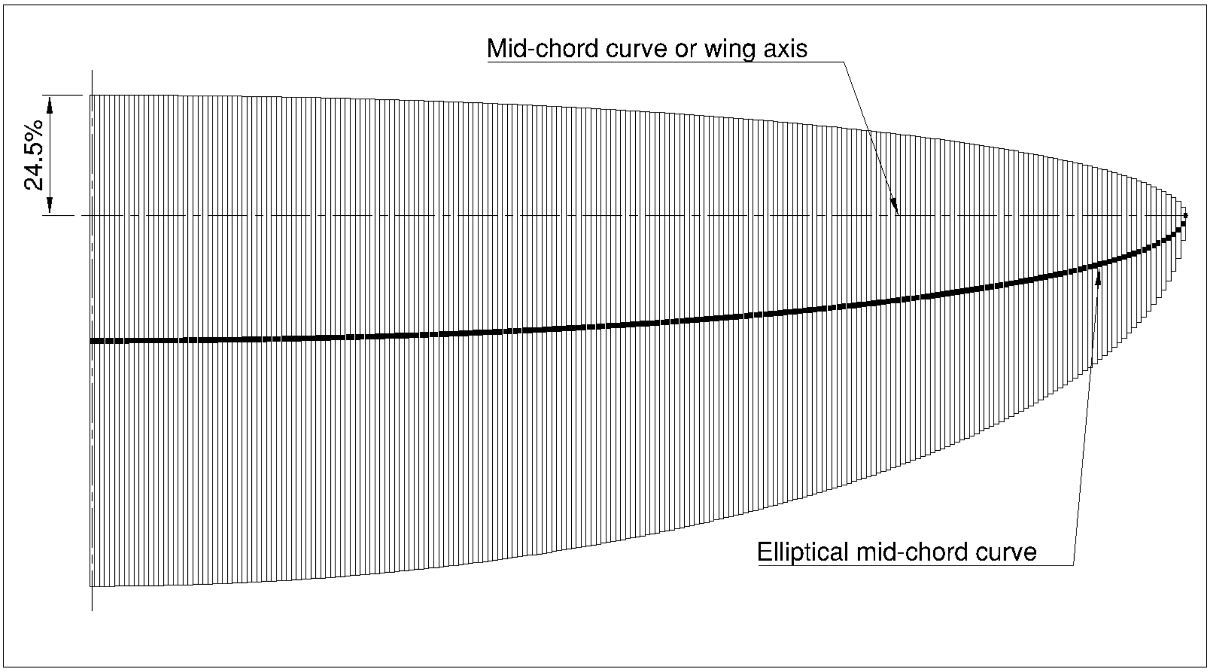

Figure 9

In figure 9 the pin centres are positioned along an elliptical curve. As long as the axis of this elliptical curve is horizontal (perpendicular to the root chord), both leading the trailing edges of the wing outline will themselves be ellipses.

Here the wing planform is defined by two semi-ellipses. The leading edge and mid-chord are symmetrical about the wing axis, while the trailing edge chords are symmetrical about the mid-chord.

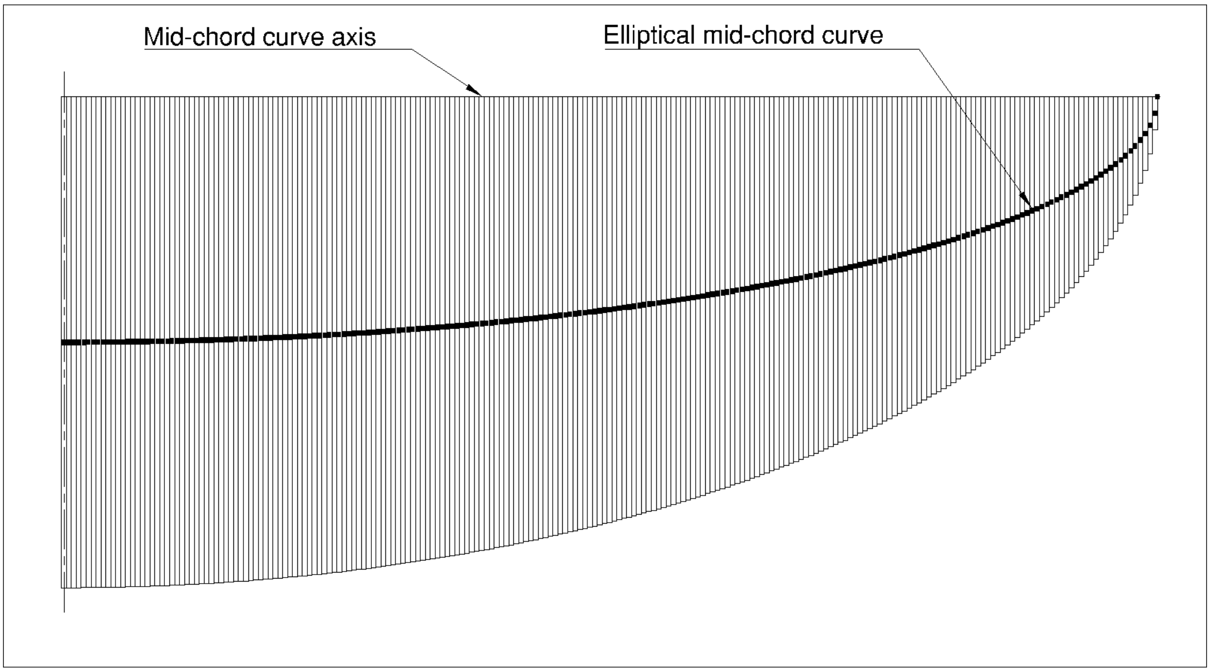

Figure 10

Fig. 10 shows an elliptic wing plan-form with a straight leading edge. The mid-chord and trailing edge are semi-ellipses.

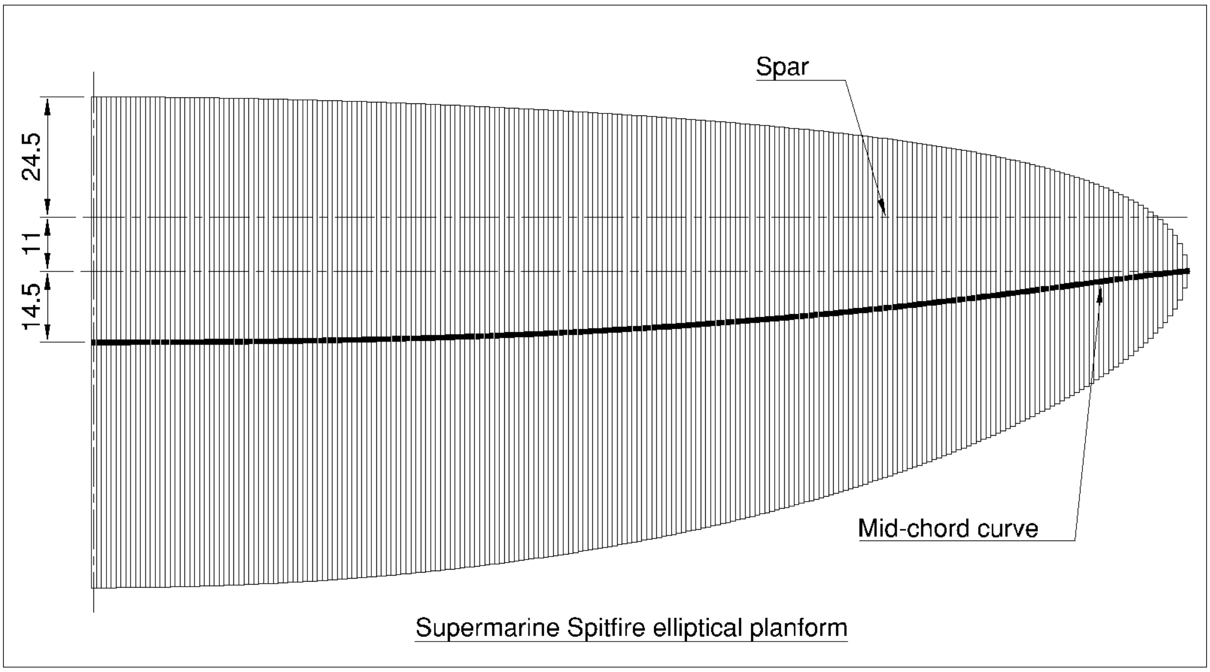

Figure 11

Fig. 11 shows the Spitfire wing plan-form. Here, the mid-chord line is no longer an ellipse and neither the leading edge nor the trailing edge are elliptical, even though the wing planform has a perfectly elliptical chord distribution. This platform, therefore, is by definition a true elliptic wing.

The wing-tip is raised 14.5 inches with a somewhat pronounced leading edge in front of the spar for increased strength, while curving the trailing edge into an elegant tip. The mid-chord curve has its ends blending horizontally tangent so that the wing looks symmetrical both at the root as well as at the tip. Aesthetically, the wing-tip is particularly pleasing to the eye when compared to Fig. 9. With its single radius it gives the impression of a single ellipse, while the latter is composite of two different radii. Wing-tip radius is identical to one of a symmetrical ellipse as shown Fig. 8 and is defined as

$$r_{tip} = \frac{50^2}{222.5}$$

Equation 3

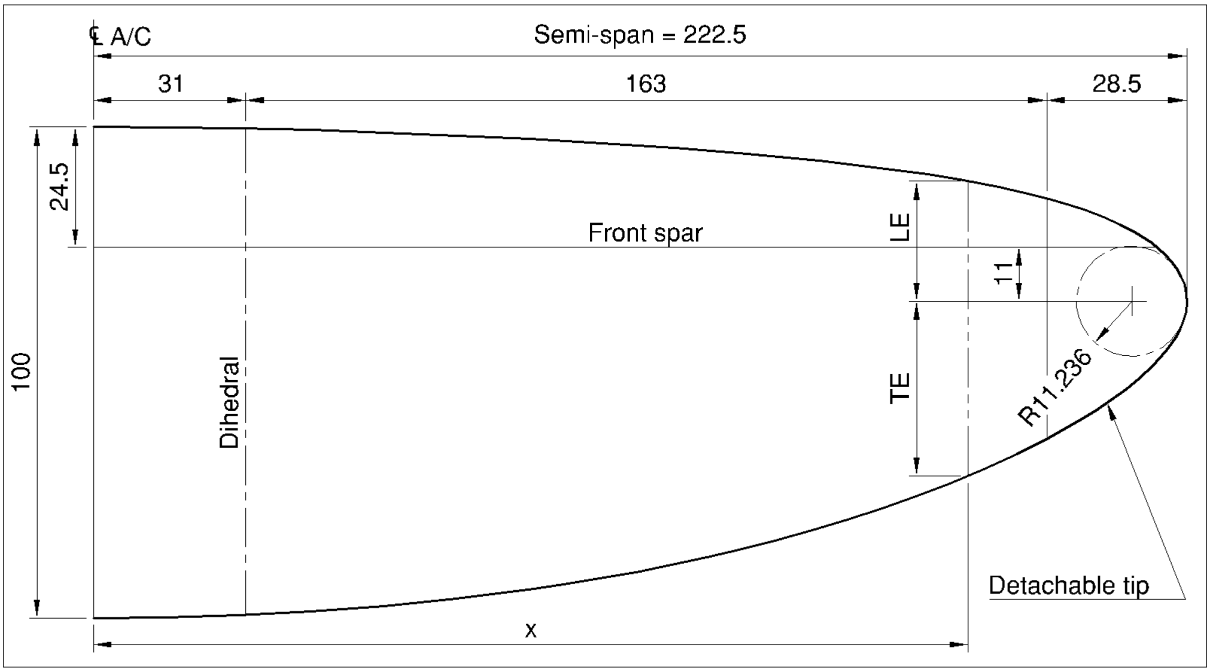

Figure 12

Figure 12 shows the Spitfire wing main dimensions. Dihedral is flattened and station dimensions are along the main-spar datum-line.

Equations for the leading edge and trailing edge shape are given below:

$$LE = \sqrt{1-\left(\frac{x}{222.5}\right)^2}\left[14.5\left[1-\left[1-\left(\frac{x}{222.5}\right)^2\right]^{0.72}\right]^{1.57}+35.5\right]$$

Equation 4

$$TE = \sqrt{1-\left(\frac{x}{222.5}\right)^2}\left[14.5\left[1-\left[1-\left(\frac{x}{222.5}\right)^2\right]^{0.72}\right]^{1.57}-64.5\right]$$

Equation 5

Both curves are vertical offsets from wing-tip radius centre. Their derivation is given in the appendix.

Equation 1 for Chord distribution is still valid as well as all the others given for the planform in Fig. 8. Alternatively therefore, either Equation 4 or Equation 5 can be used in conjunction with Eq. 1 to define the planform.

This is what the Spitfire wing might have looked like had the Supermarine team had ample time to re-loft the aerofoil sections during last minute changes in design that were made and the resulting planform. As a result, existing lofted aerofoil sections done for a different platform were cleverly arranged to conform to an accurate elliptical distribution over most of the span and slightly deviating from the ideal at the tips. This is hardly noticeable and is discussed in part one of this study.

This deviation is shown in Figs. 13 & 14.

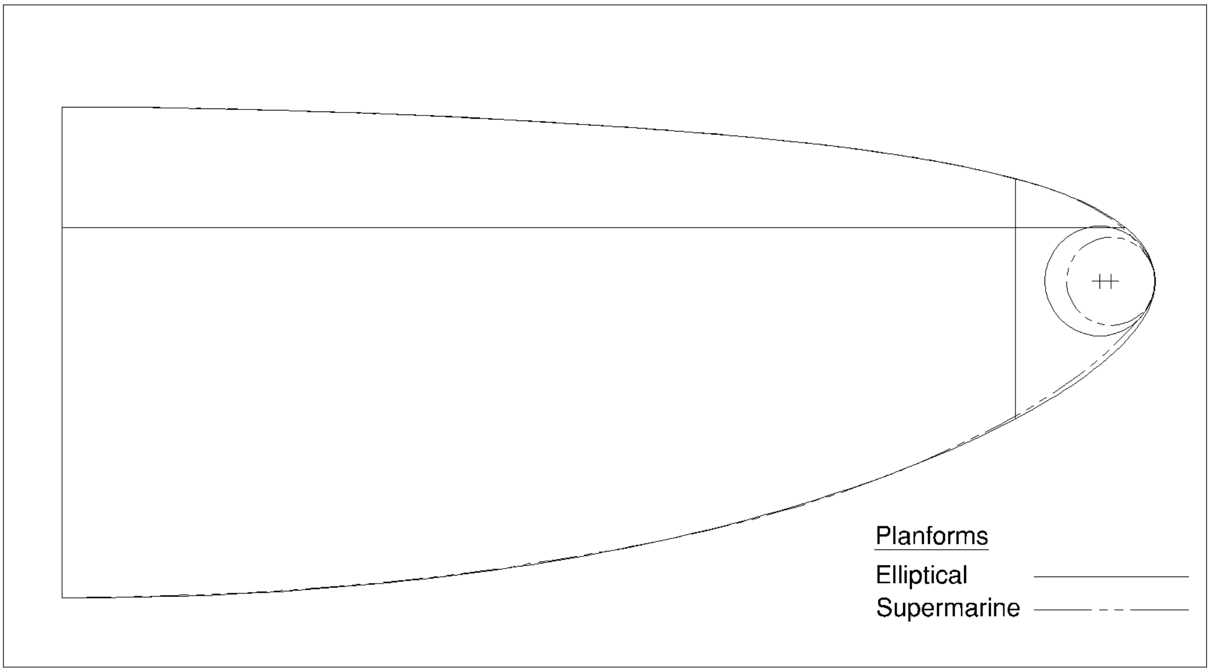

Figure 13

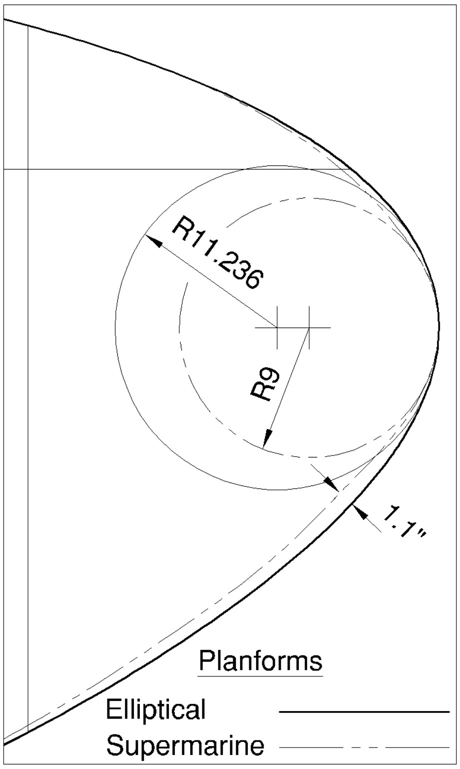

Figure 14

The difference in the two wing planforms is at a maximum of about one inch located at the trailing edge and some 6" inboard from the tip (see Fig. 14).

Many other planform geometry variations are possible as shown in Figs 15 to 17. While none have ellipses describing their shape, they all have perfectly elliptical chord distribution along their span.

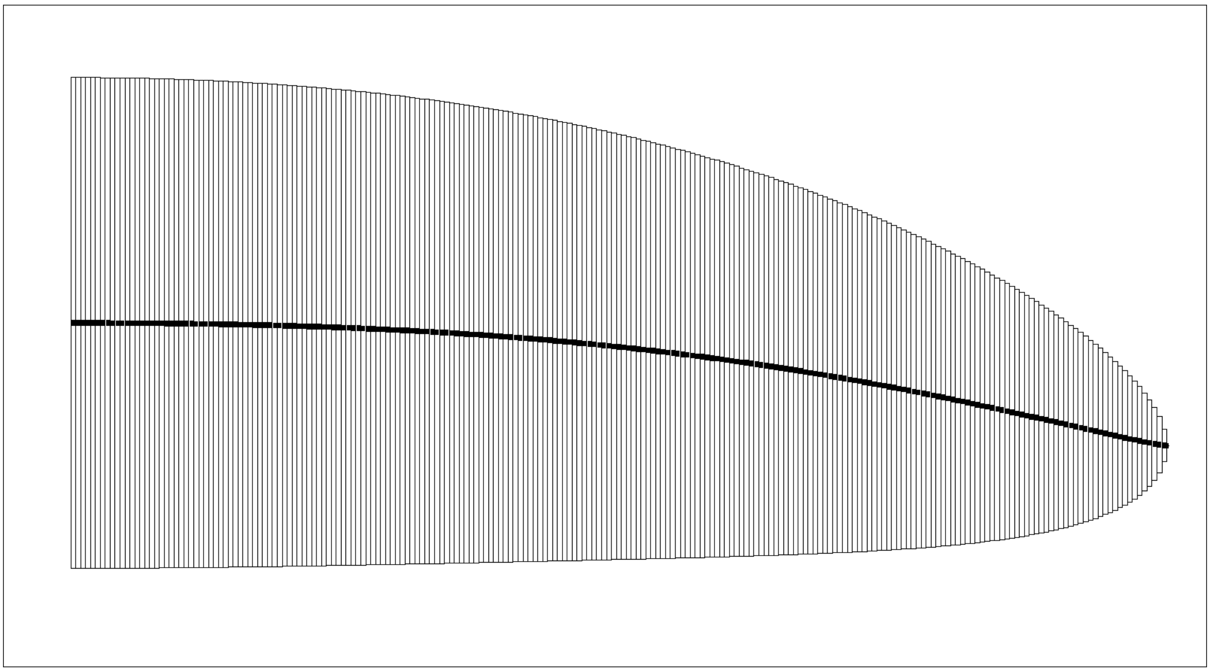

Figure 15

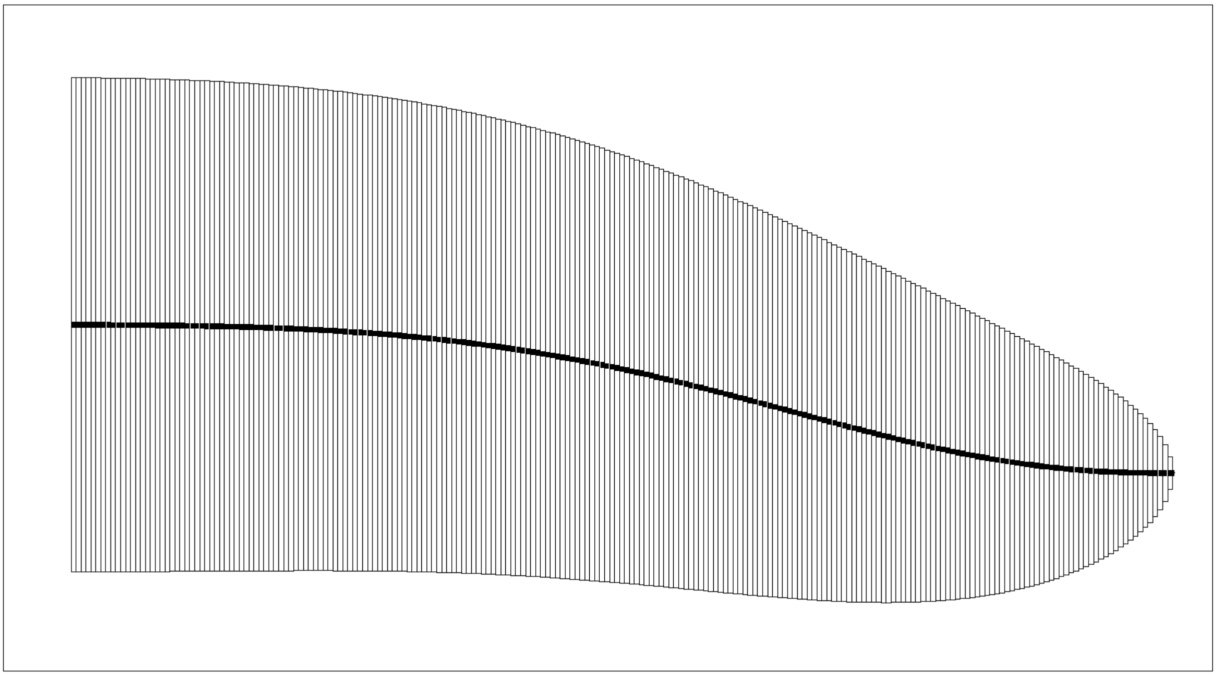

Figure 16

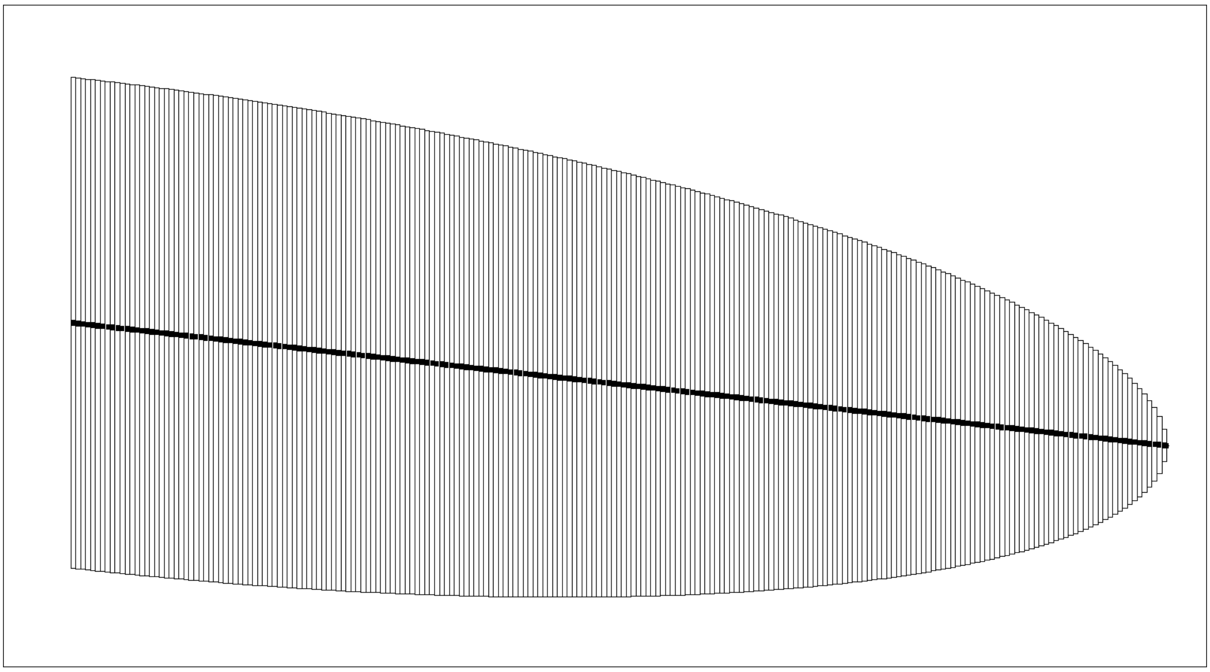

Figure 17

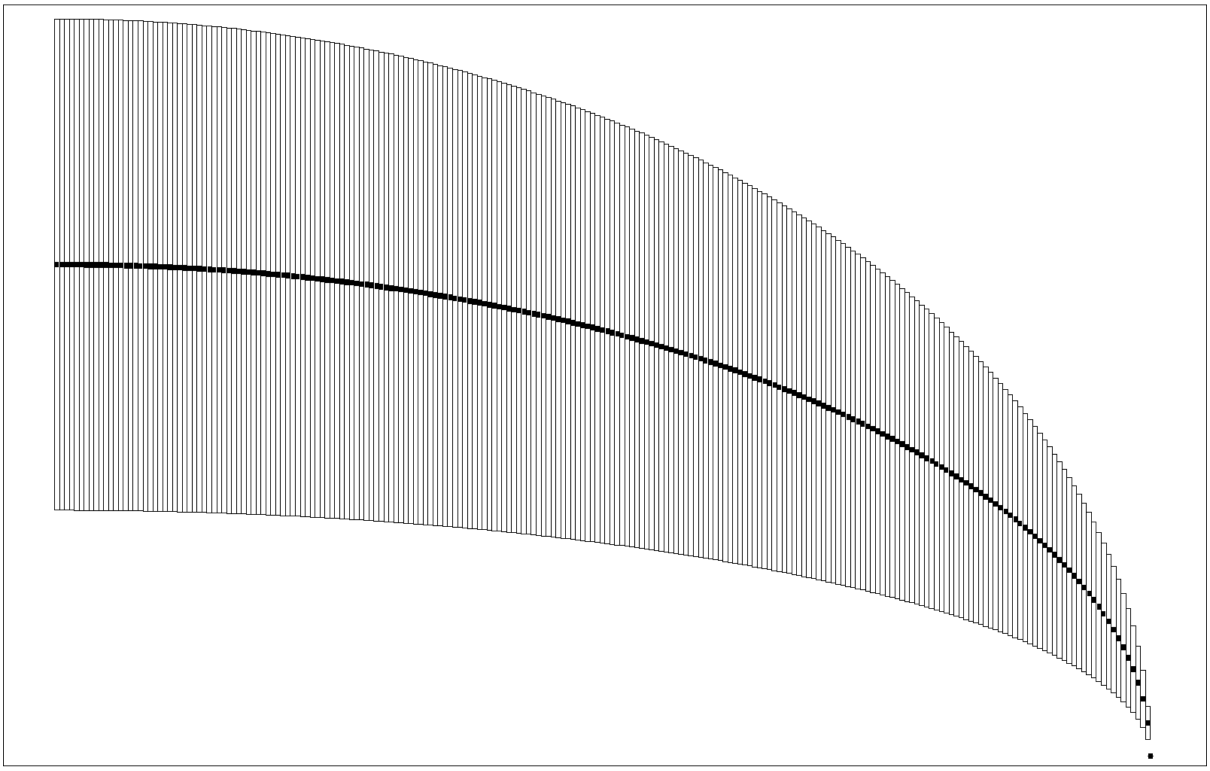

Figure 18

Figure 18 is another extreme example, but since the mid-chords are arranged along an ellipse, both leading and trailing edges are also ellipses.

Perhaps it should also be pointed out that leading and trailing edges may be swapped so that, for example, the wing in figure 17 can either have its leading edge at the top or at the bottom, depending on whether the designer requires a rearward or a forward wing-seep.

Appendix

Derivation of equations for Spitfire-type elliptic wing-planform

The pleasing shape of the Supermarine Spitfire wings can be attributed to span-wise chord rearrangement of its mono-elliptic planform.

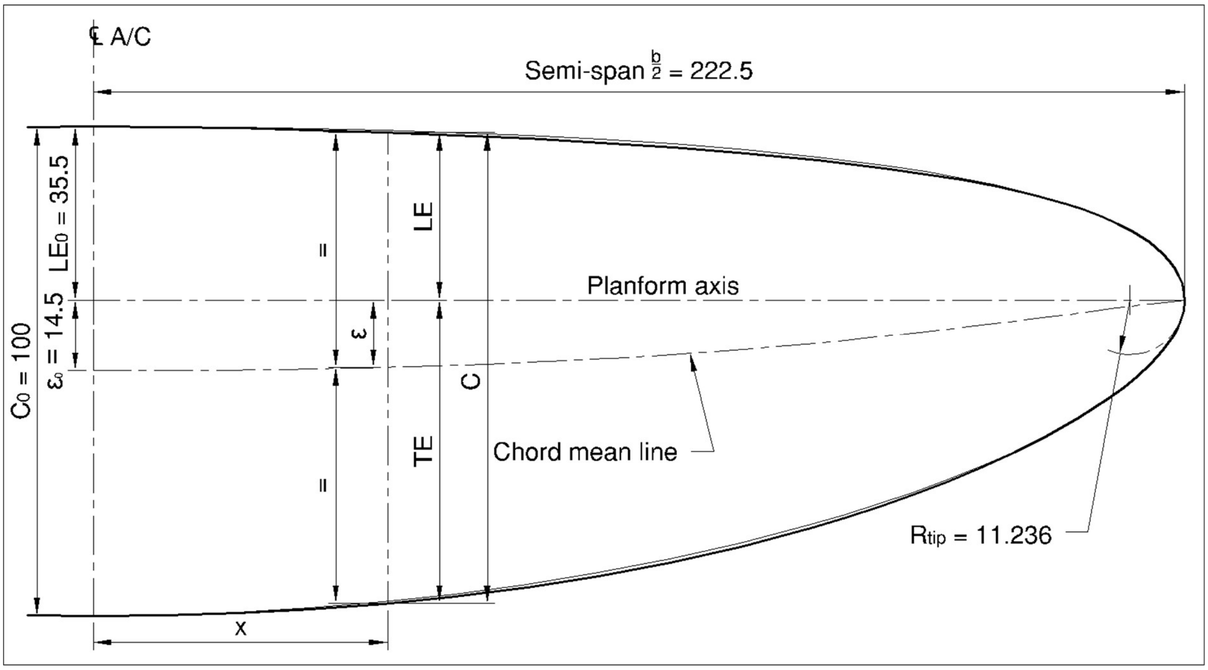

Figure 19

The normally symmetric ellipse has the tips shifted towards the leading edge by 14.5 inches.

$$\frac{\varepsilon_0}{C_0} = 0.145$$

Equation 6

It is convenient to denote this chord-wise shift position as the 'planform axis as the leading edge offsets on the Supermarine drawings use the front spar datum line which is parallel and 11 inches away towards the leading edge. As a result the leading edge offsets (LE) will be numerically 11" greater than offsets 'A' given on Supermarine drawing No. 33708 Sheet 8 titled "WING GEOMETRY".

By studying Figure 19 it will be evident that local leading edge offset to chord ratio

$$\lambda = \frac{LE}{C}$$

Equation 7

will vary along the semi-span from $\lambda_0 = 0.355$ at root to $\lambda_t = ½$ at tip.

The straight line in diagram 3 shows this simple relation.

This relation for the Supermarine Spitfire wing is shown in diagram. 1 (see part one).

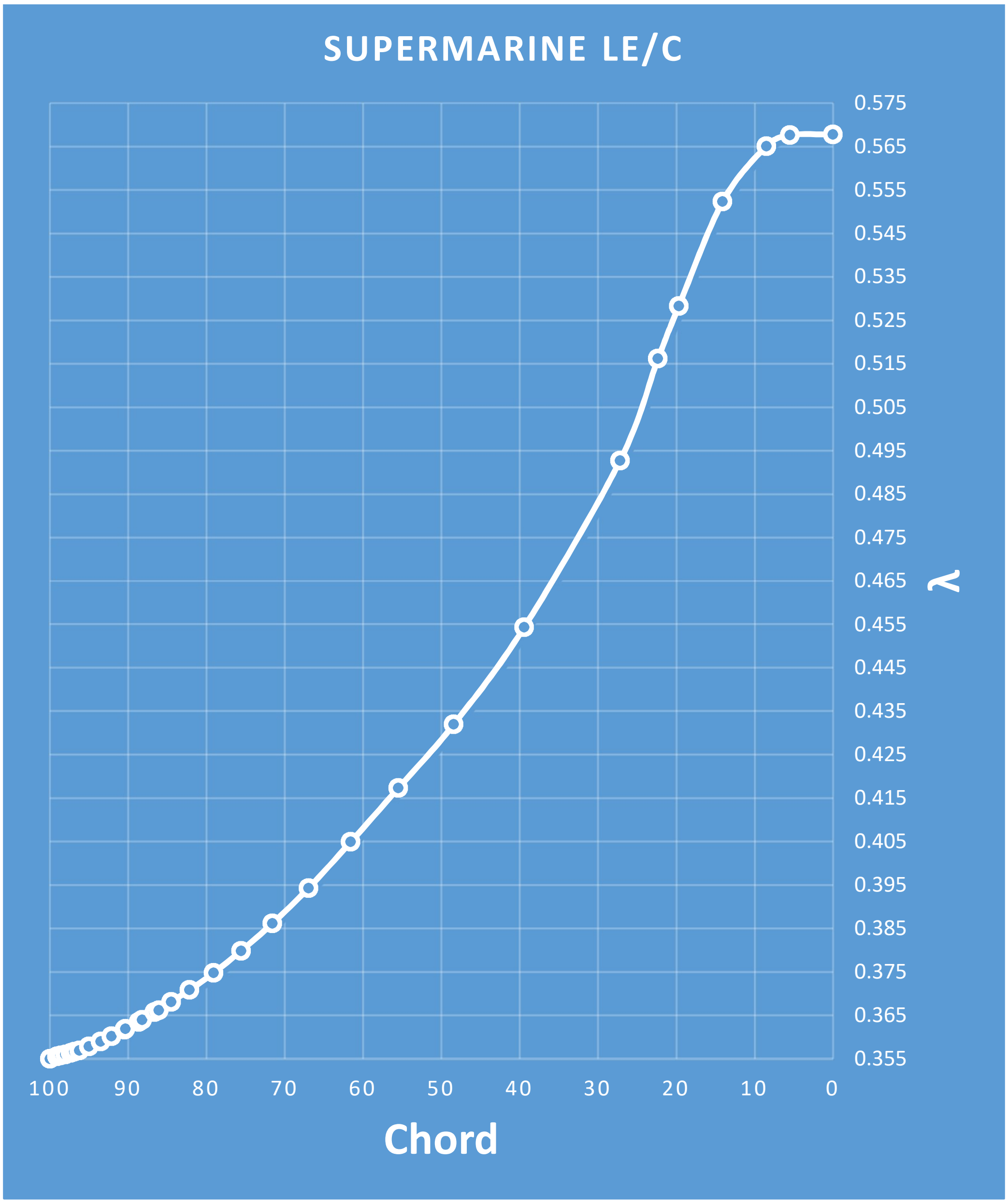

Diagram 11/2

Since the centre of the radius of curvature at the wing-tip lies on the platform axis it follows that the leading edge and trailing edge points are equidistant at the tip and therefore this ratio must equal to ½.

Diagram 1 shows that this this ratio is greater than ½ and equals 0.5678. This over bulge of the leading edge at the tip is due to slightly shorter chords than that of elliptical distribution. However, the shape of the curve indicates gradual transitions especially at the tip where the curve's slope tends to zero. This ensures that the mono-elliptic tip gradually distorts towards the root, ensuring an aesthetically pleasing appearance.

The slope at root has a positive value but the effect is far less noticeable where the ellipse's leading and trailing edges have zero slopes.

This 'S' curve can be generated in various ways but a simple equation with enough flexibility is desirable. After considering a few options a super-quadratic was chosen as the two exponents offer adequate flexibility for an excellent fit in the following form:

$$y = \left(1 - x^m\right)^n$$

Equation 8

Leading edge distance to chord relation may therefore be expressed as

$$\lambda(C) = \left(\lambda_t - \lambda_0\right)\left[1 - \left(\frac{C}{C_0}\right)^m\right]^n + \lambda_0$$

Equation 9

For best fit to the Spitfire planform and at a setting of $\lambda_t = ½$ at the tip, the above exponents were calculated to be $m = 1.44$ and $n = 1.57$.

The ratio λ is shown in Diagram 2

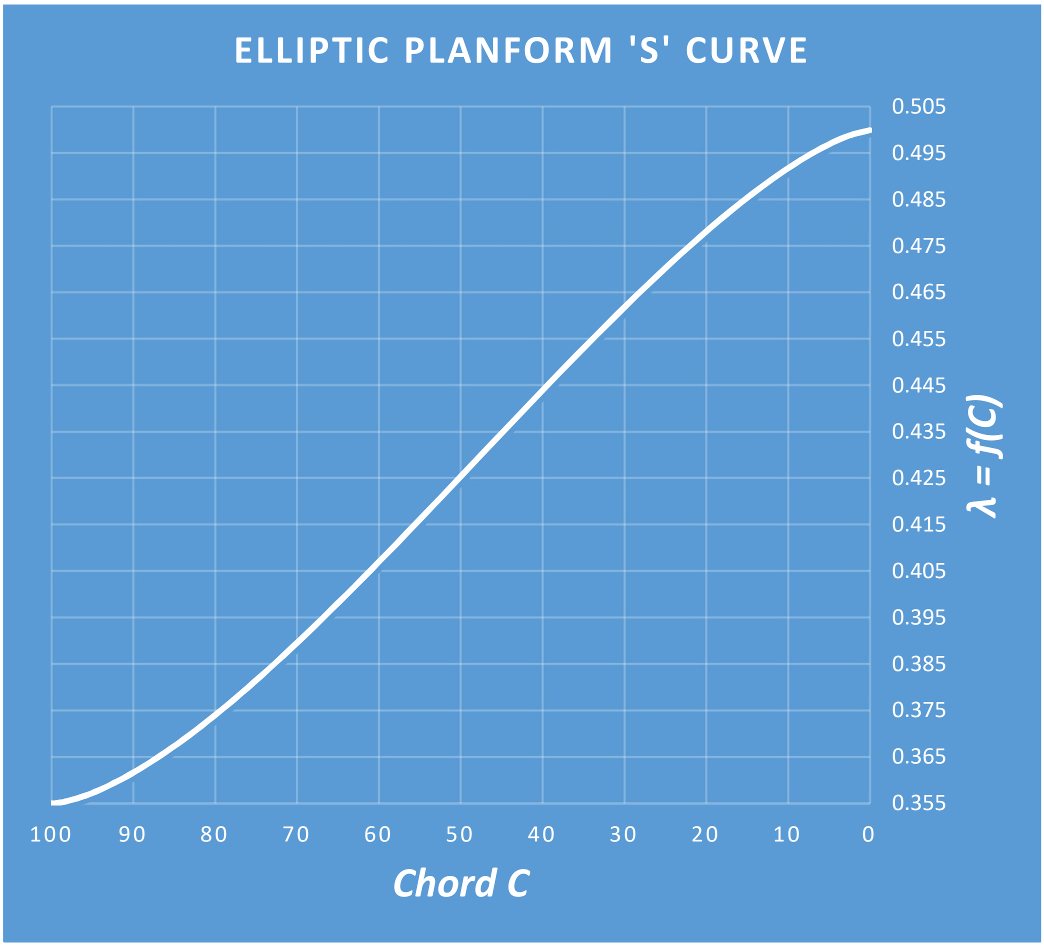

Diagram 2

Local chord for an elliptical wing is given by

$$C(x) = C_0\sqrt{1-4\left(\frac{x}{b}\right)^2}$$

Equation 10

The leading edge from Equation 7 is

$$LE = C\lambda$$

Equation 11

Substituting Equation 10 and Equation 9 into Equation 11 we get

$$LE(x) = C(x)\left[\left(\lambda_t - \lambda_0\right)\left[1-\left(\frac{C(x)}{C_0}\right)^m\right]^n + \lambda_0\right]$$

Equation 12

And similarly by subtracting Equation 12 from Equation 10

$$TE(x) = C(x)\left[\left(\lambda_t - \lambda_0\right)\left[1-\left(\frac{C(x)}{C_0}\right)^m\right]^n + \lambda_0 - 1\right]$$

Equation 13

where exponents m and n may be varied, C_0 is the root chord and b is the wing-span. A whole family of shapes are possible by varying the exponents.

For the Spitfire planform having an 'S'-type curve

$$y(x)_{LE,TE} = 100\sqrt{1-\frac{4x^2}{445^2}}\left[0.145\left[\left[1-\left(1-\frac{4x^2}{445^2}\right)^{\frac{1.44}{2}}\right]^{1.57} - 1\right] \pm \frac{1}{2}\right]$$

Equation 14

The planform mean-line $\varepsilon = f(x)$ can be expressed as

$$\varepsilon(x) = LE(x) - \frac{C(x)}{2}$$

Equation 15

Alternatively

$$\varepsilon(x) = C(x)\left(\lambda(x) - \frac{1}{2}\right)$$

Equation 16

Substituting we get

$$\varepsilon(x) = C(x)\left[\left[\left(\lambda_t - \lambda_0\right)\left[1 - \left(\frac{C(x)}{C_0}\right)^m\right]^n + \lambda_0\right] - \frac{1}{2}\right]$$

Equation 17

For the Spitfire wing the mean line ε is

$$\varepsilon(x) = 14.5\sqrt{1 - \frac{4x^2}{445^2}}\left[\left[1 - \left(1 - \frac{4x^2}{445^2}\right)^{\frac{1.44}{2}}\right]^{1.57} - 1\right]$$

Equation 18

Elliptical Planform With Linear λ

For a linear relation of LE/C, the exponents m and n are unity so that

$$\lambda(C) = \left(\lambda_0 - \lambda_t\right) - \frac{C}{C_0} + \lambda_t$$

Equation 19

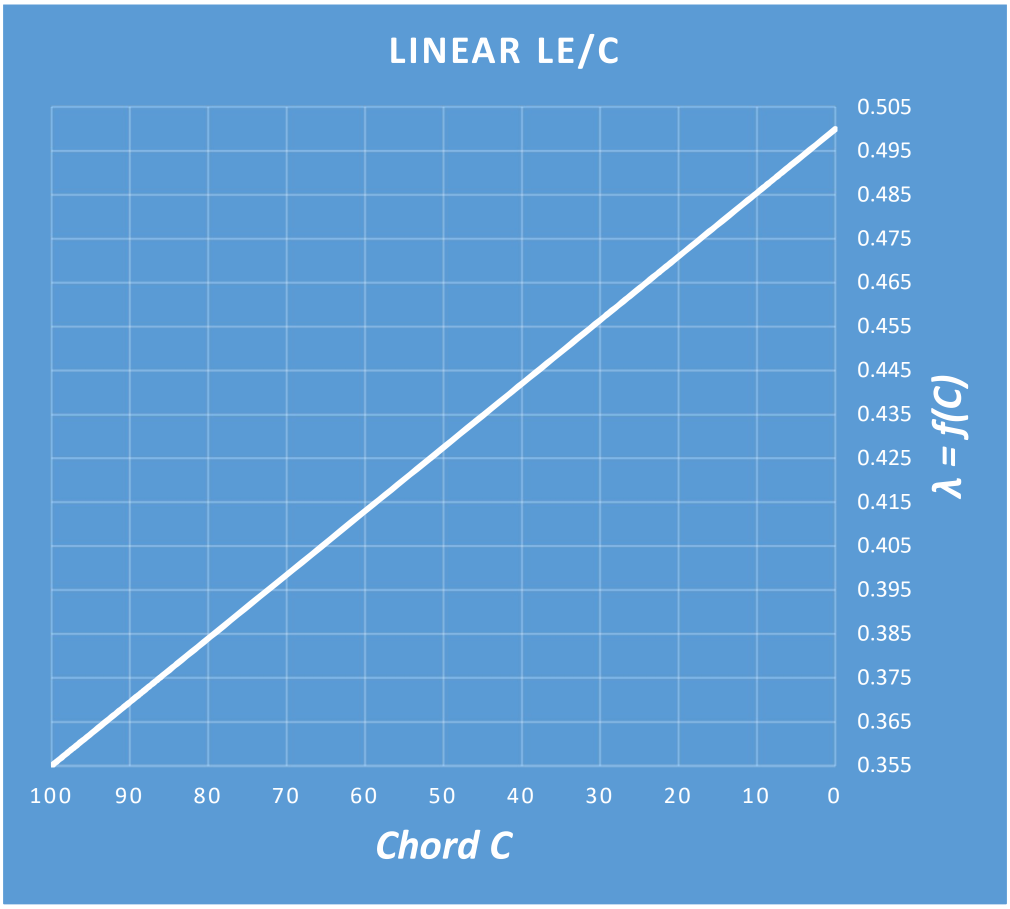

Diagram 3

In this case Equation 13 and Equation 14 reduce to

$$LE(x) = C(x)\left[(\lambda_0 - \lambda_t) - \frac{C(x)}{C_0} + \lambda_t\right]$$

Equation 20

$$TE(x) = C(x)\left[(\lambda_0 - \lambda_t) - \frac{C(x)}{C_0} - \lambda_t\right]$$

Equation 21

This planform is shown in Figure 20.

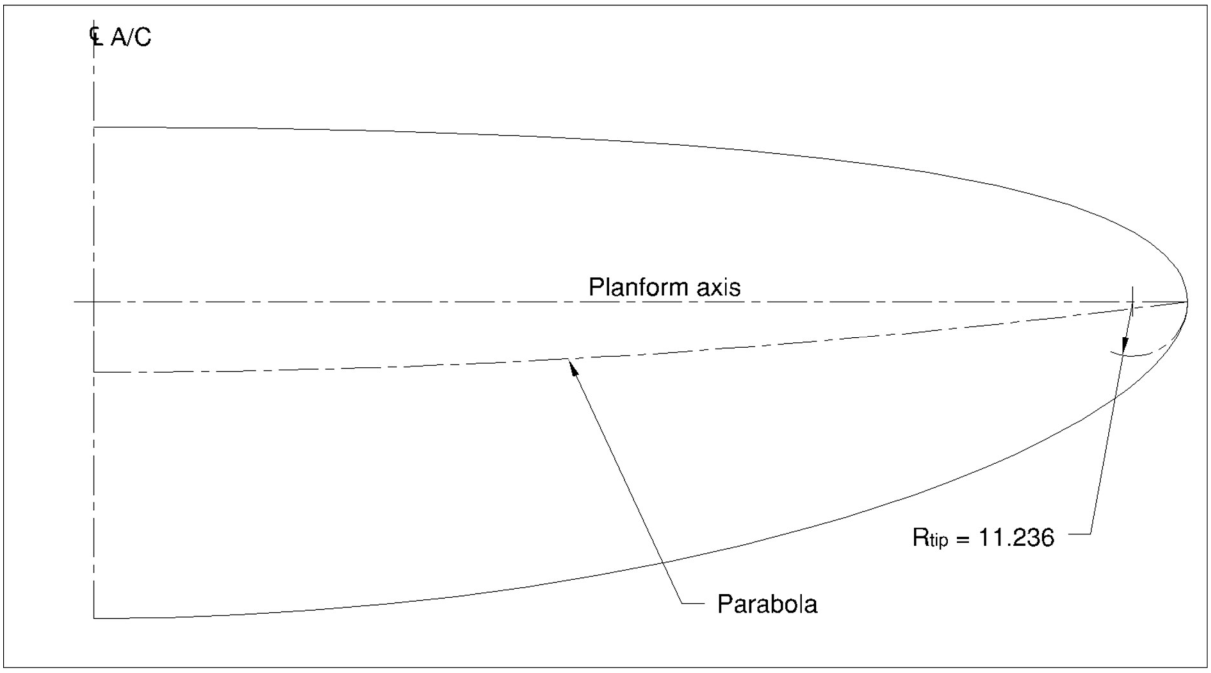

Figure 20

It is evident form Figure 20 that a linear λ produces just as elegant planforms as does the 'S' curve. The mean-line of the planform is a parabola and its equation is

$$\varepsilon(x) = C(x)\left[\left[(\lambda_0 - \lambda_t) - \frac{C(x)}{C_0} + \lambda_t\right] - \frac{1}{2}\right]$$

Equation 22

Simplifying and putting $\varepsilon_0 = C_0(\lambda_t - \lambda_0)$

$$\varepsilon(x) = \varepsilon_0\left(4\left(\frac{x}{b}\right)^2 - 1\right)$$

Equation 23

In this case the planform may also be expressed as

$$y(x)_{LE,TE} = \varepsilon_0 \left(4\left(\frac{x}{b}\right)^2 - 1 \right) \pm \frac{C(x)}{2} \qquad \text{or simply} \qquad y(x)_{LE,TE} = -\varepsilon_0 \left(\frac{C(x)}{C_0}\right)^2 \pm \frac{C(x)}{2}$$

Equation 24

Equation 25

This is identical to Equation 20 & Equation 21. Interesting planforms may be obtained by varying ε_0 over a wide range. It may also have values beyond C_0.

A comparison is given in Figure 21 with Figure 20 superimposed on Figure 19. The 'straight line' λ leading edge is 0.8 inches ahead of 'S' curve λ (or the Spitfire planform). This minor deviation might well be justified by the simpler equations (Equation 20 & Equation 21).

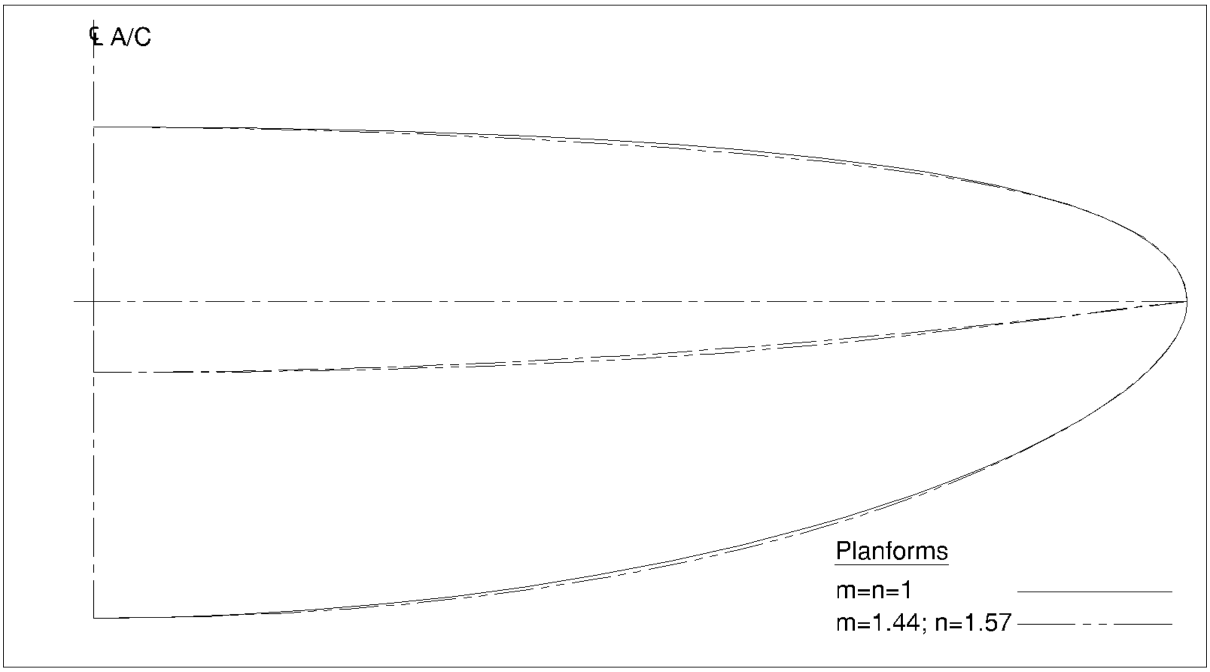

Figure 21

Another 'S' curve may be represented by a trigonometric relation:

$$\lambda(C) = \frac{\varepsilon_0}{C_0}\left(\cos\left(\frac{\pi C}{2C_0}\right)^m - 1 \right) + \frac{1}{2}$$

Equation 26

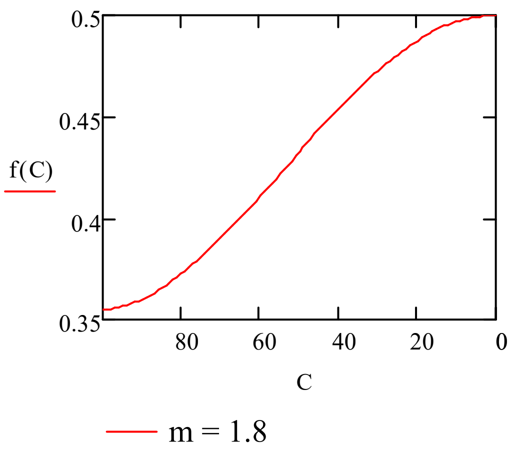

Diagram 4

Where exponent m is the shape factor and affects mostly the root sections while the curve is tangent to the horizontal axis at the tip.

A fairly close approximation for the Spitfire wing planform is obtained when exponent m is 1.8

$$\lambda = 0.355 + 0.145\cos\left(\frac{\pi C}{200}\right)^{1.8}$$

Equation 27

Combined with Equation 10 the leading and trailing edges are given by

$$LE = \frac{29}{890}\sqrt{445^2 - 4x^2}\left(\cos\left(\frac{\pi}{890}\sqrt{445^2 - 4x^2}\right)^{1.8} + 71\right)$$

Equation 28

$$TE = \frac{29}{890}\sqrt{445^2 - 4x^2}\left(\cos\left(\frac{\pi}{890}\sqrt{445^2 - 4x^2}\right)^{1.8} - 129\right)$$

Equation 29

This planform has a slightly less pronounced leading edge and at Station 110” from root chord is 0.16” less than the Spitfire one.

Additional properties of elliptic wing planforms

Mean aerodynamic chord (MAC)[2] is given by

$$\overline{C}_a = C_{root}\frac{8}{3\pi}$$

Equation 30

[2] This is an imaginary chord of an equivalent rectangular wing with the same area and with equal aerodynamic moments as the actual wing.

And equals 84.88”.

The mean aerodynamic chord is located at station

$$x_a = \frac{2 \cdot 445}{3\pi}$$

Equation 31

And equals 94.43”.

Thickness distribution

A virtual wing of a trapezoidal planform may be used for distributing aerofoil thickness along the span and may be defined as follows:

λ = Taper ratio of the trapezoidal planform (tip chord where root chord may be unity).

b = Wing span

T_0 = Root aerofoil thickness in percent of root chord

T_t = Tip aerofoil thickness in percent of tip chord

The elliptic wing thickness distribution along the spar is

$$T(x) = \frac{T_0 \cdot b - 2x\left(T_0 - T_t \cdot \lambda\right)}{b + 2x\left(\lambda - 1\right)}$$

Equation 32

For any station x from centre line of aircraft.

If the taper ratio λ = 0.5, span b = 445”, T_0 = 13% and T_t = 6% Equation 32 simplifies to

$$T(x) = \frac{3115}{x - 445} + 20$$

Equation 33

- End -